Trigonometry 2

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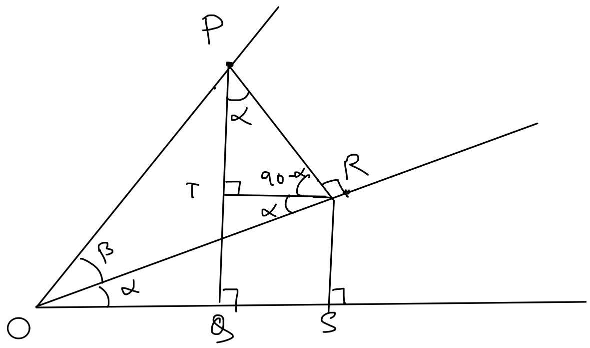
$$\sin(\alpha+\beta) = \frac{PQ}{OP}$$

$$\sin(\alpha) = \frac{RS}{OR}$$

$$\sin(\beta) = \frac{PR}{OP}$$



$PQ \perp OS$

$PR \perp OR$

$\angle QPR = \alpha$

$\angle TRO = \alpha$

TRSQ is a rectangle

$\Rightarrow TQ = RS$

$$\sin(\alpha+\beta) = \frac{PQ}{OP}$$

$$= \frac{PT+TQ}{OP} = \frac{PT+RS}{OP} = \frac{PT}{OP} + \frac{RS}{OP}$$

$$= \frac{PT}{PR} \times \frac{PR}{OP} + \frac{RS}{OR} \times \frac{OR}{OP} = \boxed{\cos\alpha \sin\beta + \sin\alpha \cos\beta}$$

$$\cos^2(\alpha+\beta) = 1 - \sin^2(\alpha+\beta) = 1 - \cos^2\alpha \sin^2\beta - 2\cos\alpha\cos\beta\sin\alpha\sin\beta - \sin^2\alpha\cos^2\beta$$

$$= \cos^2\alpha + \sin^2\alpha - \cos^2\alpha\sin^2\beta - 2\cos\alpha\cos\beta\sin\alpha\sin\beta - \sin^2\alpha\cos^2\beta$$

$$= \cos^2\alpha(1-\sin^2\beta) + \sin^2\alpha(1-\cos^2\beta) - 2\cos\alpha\cos\beta\sin\alpha\sin\beta$$

$$= \cos^2\alpha\cos^2\beta - 2\cos\alpha\cos\beta\sin\alpha\sin\beta + \sin^2\alpha\sin^2\beta$$

This means both the solutions are satisfiable here

$$\cos(\alpha+\beta) = \pm(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

$\Rightarrow$ But one of them will be true just as in

$$\boxed{\begin{array}{ll} x^2 = y^2 & x^2 = y^2 \mid x > 0 \\ \Rightarrow x = \pm y & \Rightarrow x = |y| \end{array}}$$

The give condition for $\cos(\alpha+\beta)$ can be checked by other techniques

Homework Find the value of $\cos(\alpha+\beta)$ using the same method above as in $\sin(\alpha+\beta)$. Then check for $\sin(\alpha-\beta)$ and $\cos(\alpha-\beta)$